

# A Novel Optimal Control Design for Reducing of Time Delay Effects in Teleoperation Systems

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**Abstract**—The purpose of designing a controller for a teleoperation system is achieving stability and high efficiency in presence of factors such as time delay, noise, system disturbance and modeling errors. In this article seven new schemes for teleoperation systems is suggested to reduce the error of tracking between the master and the slave systems. In the first and second structures, optimal control is applied to the master and in the third structure it is applied to the slave. In the forth structure, optimal controller has been designed in both the master and slave subsystems and by a suitable combination of the output signals of both controllers and exerting it to slave, it has been tried to create the best performance with regard to tracking. In the 5th structure, optimal controller is applied to both the master and slave systems and the output of each controller is then applied to its own system. In the structures 6 and 7, using optimal control and wave transformation, it has been tried to reduce the tracking error between the master and the slave systems which is considered as the cost function in this article. In these two structures, optimal controller guarantees the efficiency of the system and wave variables method has been used to guarantee the stability of the system for constant time delay. In this method, the appropriate selection of weighting factors of cost function and wave impedance plays a decisive role in the improvement of system performance and reduction of tracking error between the master and the slave.

**Keyword-** teleoperation system; wave variables method; optimal controller; tracking error; cost function

## I. INTRODUCTION

A teleoperation system enables human operator to implement given task in a remote manner or enhance his/her capability to handle both the micro and the macro worlds. A typical teleoperation system consists of a local master manipulator (master site) and a remotely located slave manipulator (slave site). The human operator controls the local master manipulator to drive the slave one to implement a given task remotely. The system must be completely "transparent" so that the human operator can feel as if she/he were able to manipulate the remote environment directly. In general, in the design of teleoperation systems there is a trade off between high transparency and sufficient stability

margins [1]. The main control strategies proposed for bilateral teleoperation systems with constant delay include the delay compensation proposed by Anderson and Spong [2], which assures the asymptotic stability by Niemeyer and Slotine [3], where wave transformations are used to keep the passivity of the communication channel. Remote compliance control proposed by Kim et al. [4], where the remote compliance smoothes the interaction between the slave and the environment, but the transparency is poor for delays greater than 1 s. The internet has time-varying characteristics (for example, the amplitude of time delay), therefore it is useful to use information-measured in-line of communication channel to design the control structure, as in [5,6]. Where these delay compensations are based on the wave variables modifying any main parameter (apparent damping or gain, respectively) depending on the time delay. The control structures based on wave variables keep the passivity with delay impedance, but at the expense of reducing system transparency. The paper is organized as follows: in section 2 wave variables method is introduced and in section 3 the optimal control design used in this article is presented. In section 4 simulations of different structures for different weighting factors is demonstrated and the validity of this scheme is established. In section 5 the results are discussed.

## II. WAVE VARIABLES METHOD

Wave variables method presents a modification or extension to the theory of passivity, which creates robustness to arbitrary time delay. This method is also closely related to the scattering and small gain theories [7].

In this sense, we define  $U$  to denote the forward or right moving wave, while  $V$  denotes the backward or left moving wave.

The wave variables  $(U, V)$  can be computed from the standard power variables  $(\dot{X}, F)$  by the following transformation.

$$U = \frac{b\dot{X} + F}{\sqrt{2b}}, V = \frac{b\dot{X} - F}{\sqrt{2b}} \quad (1)$$

Where  $b$  is the characteristic wave impedance and may be a positive constant or a symmetric positive definite matrix. Fig. 1 shows the basic wave transformation.

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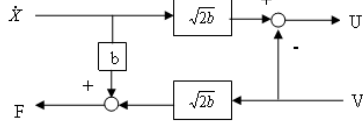


Fig. 1: the basic wave transformation

using the velocity  $\dot{X}$  and left moving wave  $V$  as the inputs, the transformation will determine the force  $F$  and right moving wave  $U$ . All other combinations are also possible.

$$F = b\dot{X} - \sqrt{2b}V, U = -V + \sqrt{2b}\dot{X} \quad (2)$$

Fig. 2 shows wave variables transformation used in this article.

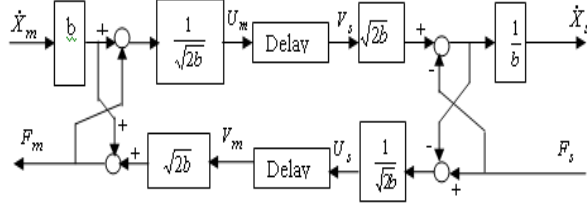


Fig. 2: wave based communication, by transforming velocity-force variables (subscripts s and m denote slave and master respectively)

In this configuration, both velocity and force variables are first transformed into wave variables before transmission. Upon arrival information (velocity and force) is extracted from the slave variables, in accordance with the convention used by Niemeyer [8], right moving waves are defined as positive and left moving waves are negative. The full details about wave variables method is described in [7].

### III. OPTIMAL CONTROL DESIGN IN TRACKING PROBLEMS

Continuous time linear system is presented in the state space like [9]:

$$\dot{X}(t) = AX(t) + Bu(t) \quad (3)$$

Cost function that should be minimized:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left( (X(t) - r(t))^T Q (X(t) - r(t)) + u^T(t) R u(t) \right) dt \quad (4)$$

The final  $t_f$  is fixed.  $H$  and  $Q$  are real symmetric positive semi-definite matrices and  $R$  is real symmetric positive definite matrix. It is supposed that the controls and positions are not bounded and  $x(t_f)$  is free. The acceptable control  $u(t)$  that causes the equation system of (3) to follow the desired route of  $r(t)$  and minimize the cost function is as follows:

$$u(t) = -R^{-1}B^T K(t)X(t) - R^{-1}B^T S(t) \quad (5)$$

$$= F(t)r(t) + V(t)$$

In which  $F(t)$  is the feedback reinforcement matrix and  $V(t)$  is the command signal. It should be noted that  $V(t)$  depends on the system parameters and the desired route  $r(t)$ . In fact  $V(t)$  depends on the future amounts of the reference signal.

$K(t)$  and  $S(t)$  in the above equations are:

$$\dot{K}(t) = -K(t)A - A^T K(t) - Q + K(t)BR^{-1}B^T K(t) \quad (6)$$

$$\dot{S}(t) = -[A^T - K(t)BR^{-1}B^T]S(t) + Qr(t) \quad (7)$$

And boundary conditions are:

$$K(t_f) = H \quad (8)$$

$$S(t_f) = Hr(t_f) \quad (9)$$

Since in this problem the value of  $H$  is zero, as a result the boundary conditions in the equations (8) and (9) are zero.

To achieve  $u(t)$ , we should integrate equation (6) and (7)

from  $t_0$  to  $t_f$  and using boundary conditions we will find  $u(t)$ . In the first and second structure we design an optimal control in the Master system and in the third structure we design an optimal control in the slave system. The reference signal for the optimal controller in the first structure is  $X_s(t - \tau)$  and for the second structure is  $X_m(t)$  and the desired signal for the optimal controller in the third structure is  $X_m(t - \tau)$ . If we place the reference signals of the controllers in equation (6) and (7) we can find  $u(t)$  from equation (5). Figure 3 (4<sup>th</sup> structure) shows a new structure of bilateral teleoperation system with time delay in the transmission channel using combinational optimal controller.

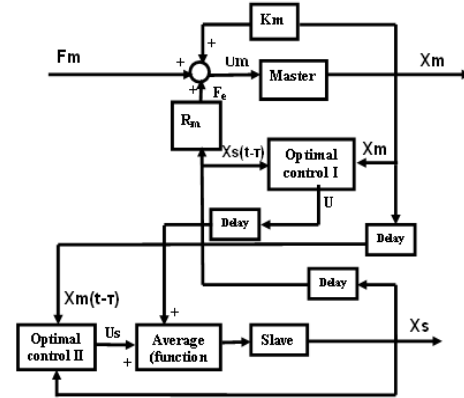


Fig. 3: structure of teleoperation system using combinational optimal controller (4th structure).

In this structure optimal controller has been designed in both the master and slave subsystems and the controlling signals of each controller have been multiplied by a certain

coefficient and are added together and the resulted controlling signal is applied to the slave in a way that it can follow the master in the least time. In this scheme we have considered only one degree of freedom for the slave and the master in the teleoperated system. The dynamic model of an element with a dof is as below:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) + mgl \sin \theta(t) = u(t) \quad (10)$$

J is the inertia of the element, M is the mass of robot manipulator, g is the gravity acceleration, l is the length of robot manipulator,  $\theta(t)$  is the rotate angle, b is the viscous friction coefficient, and u(t) is the control torque applied. The simplified linear model is:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) = u(t) \quad (11)$$

Considering the position ( $x_1(t) = \theta(t)$ ) and the velocity ( $x_2(t) = \dot{\theta}(t)$ ) as state variables, the state space representation of the master and slave is:

$$\begin{bmatrix} \dot{x}_{m1}(t) \\ \dot{x}_{m2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_m}{J_m} \end{bmatrix} \begin{bmatrix} x_{m1}(t) \\ x_{m2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix} u_m(t) \quad (12)$$

$$\begin{bmatrix} \dot{x}_{s1}(t) \\ \dot{x}_{s2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_s}{J_s} \end{bmatrix} \begin{bmatrix} x_{s1}(t) \\ x_{s2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_s} \end{bmatrix} u_s(t) \quad (13)$$

$$y_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{m1}(t) \\ x_{m2}(t) \end{bmatrix} \quad (14)$$

$$y_s(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{s1}(t) \\ x_{s2}(t) \end{bmatrix} \quad (15)$$

The structure of the other matrices that appear in the model, in this case, is the next:

$$K_m = \begin{bmatrix} k_{m1} & k_{m2} \end{bmatrix} \quad (16)$$

$$R_m = \begin{bmatrix} r_{m1} & r_{m2} \end{bmatrix} \quad (17)$$

In which  $K_m$  is the feedback matrix of the master state.  $R_m$  defines the interaction between the slave-master. It allows modeling the force reflection to the master.

In the teleoperation systems, it is necessary to take into account the remote environment. When the slave manipulator performs a task can interacting with the environment. This interaction must be considered in the design of the generated reaction forces. We have employed a simplification of the Kelvin model of the environment only by means of the stiffness ( $k_e$ ) and a viscous friction ( $b_e$ ). The reaction force that acts against the slave is given by:

$$f_s(t) = k_e \theta_s(t) + b_e \dot{\theta}_s(t) \quad (18)$$

In this way the reaction force,  $f_s(t)$ , will oppose to the slave control signal, modeling that the slave is interacting with the environment. To consider force feedback from the slave to the master, the structure of the matrix  $R_m$  must be [10]:

$$R_m(t) = \begin{bmatrix} r_{m1} & r_{m2} \end{bmatrix} = \begin{bmatrix} k_f k_e & k_f b_e \end{bmatrix} \quad (19)$$

Where  $k_f$  is the force feedback.

The master control signal, that is  $u_m(t)$ , as shown in figure (3) is:

$$u_m(t) = K_m(t)x_m(t) + R_m(t)x_s(t-T) + F_m(t) \quad (20)$$

If we place the above control signal in the state space representation of the master we have:

$$\begin{aligned} \dot{x}_m(t) &= (A_m + B_m K_m)x_m(t) \\ &+ B_m R_m x_s(t-T) + B_m F_m(t) \end{aligned} \quad (21)$$

And for the slave system, the control signal should be calculated so that the slave system can optimize the cost function of equation (4) by tracking the desired route of  $r(t)$ . the control signal of the slave system is:

$$u_s(t) = F(t)x_s(t) + V(t) \quad (22)$$

If we place the above control signal in the state space representation of the slave we have:

$$\dot{x}_s(t) = A_s x_s(t) + B_s [F(t)x_s(t) + V(t)] \quad (23)$$

In which  $V(t)$  depends on the slave system parameters and the reference signal ( $x_m(t-\tau)$ )

$F(t)$  and  $V(t)$  in the above equation are calculated from Riccati equations in equation (6) and (7).

In the 5th structure, the same as 4th structure, optimal controller is applied to both the master and slave systems but the output of each controller is then applied to it's own system. The reference signal for the optimal controller in the slave subsystem is  $x_m(t-\tau)$  and the desired signal for the optimal controller in the master system is  $x_s(t-\tau)$ . If we place the reference signals of the master and the slave systems in equation (6) and (7) we can find  $u(t)$ .

The following figure shows the 6th designed structure for teleoperation systems using wave variables and optimal control methods.

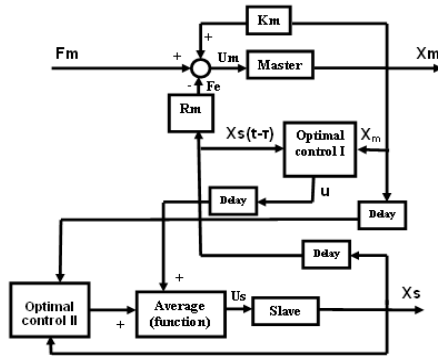


Fig. 4: Structure of teleoperation system using combinational optimal controller and wave transformation

In this structure, the position and velocity variables on the master side are transformed into wave variables. Then they pass the transmission channel and are transformed to position and velocity variables on the slave side again. The returning signal on the slave side are also first transformed into wave variables, then passing the transmission channel, wave variables are transformed into position and velocity variables on the master side. The optimal controller is used on both master and slave sides. The optimal control on the master side applies the controlling signal to the master so that the master output follows the delayed slave output. The optimal control on the slave side functions so that the slave output signal follows the delayed master output. The output of each controller is applied to its own system so that the tracking error between the master and slave is minimized.

In the 7th structure designed for teleoperation systems using optimal control methods and wave variables, an optimal controller is used on the slave side in a way that the slave output follows the delayed master output.

#### IV. SIMULATION RESULTS

The considered parameters of the master and the slave are respectively:

$$J_m = 1.5 \text{ kgm}^2, b_m = 11 \frac{\text{Nm}}{\text{rad/s}}, m_m = 28.125 \text{ kg}, l_m = 0.4 \text{ m}$$

$$J_s = 2 \text{ kgm}^2, b_s = 15 \frac{\text{Nm}}{\text{rad/s}}, m_s = 1.5 \text{ kg}, l_s = 2 \text{ m}$$

To consider the interaction between the slave and remote environment, we have considered a stiffness ( $k_e$ ) and a viscous friction ( $b_e$ ) as below:

$$b_e = 1 \frac{\text{Nm}}{\text{rad/s}}, k_e = 100 \text{ Nm/rad}$$

Considering that the force feedback gain is 0.1

$$k_f = 0.1$$

Therefore regarding the given parameters, state matrices of the slave and the master are as follows:

$$A_m = \begin{bmatrix} 0 & 1 \\ 0 & -11 \\ 1.5 & \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 \\ 1 \\ 1.5 \end{bmatrix}, C_m = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_s = \begin{bmatrix} 0 & 1 \\ 0 & -15 \\ 2 & \end{bmatrix}$$

$$B_s = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, C_s = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Given the equation (19), the value of the matrix  $R_m$  is:

$$R_m = \begin{bmatrix} k_f k_e & k_f b_e \end{bmatrix} = \begin{bmatrix} 10 & 0.1 \end{bmatrix}$$

Considering that the poles of the master are placed in the position -15 of the s plane, the following control parameters are obtained:

$$K_m = \begin{bmatrix} k_{m1} & k_{m2} \end{bmatrix} = \begin{bmatrix} -249 & -33 \end{bmatrix}$$

In this paper all of the structures are simulated for different weighting factors. The best weighting factors are achieved through trial and error.

The simulation results are demonstrated in figures 5 to 10.

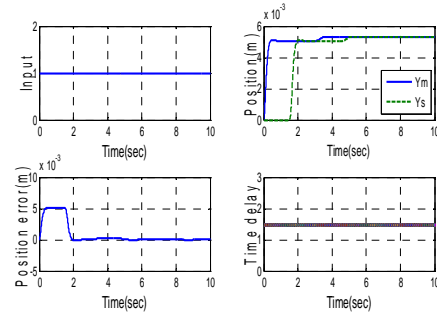


Fig. 5: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the first structure ( $T=1.5\text{s}$ )

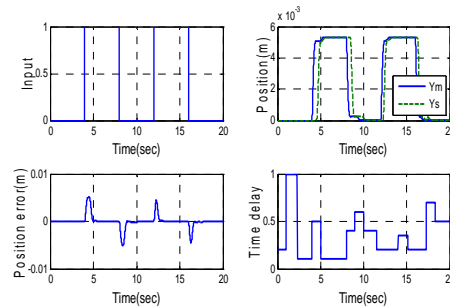


Fig. 6: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the second structure with variable time delay

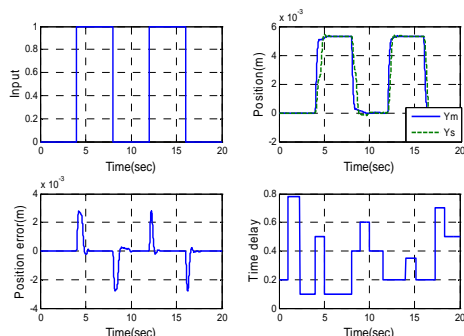


Fig. 7: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the third structure

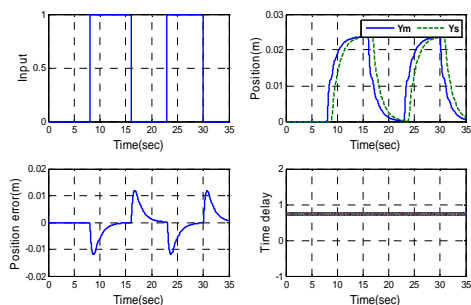


Fig. 8: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the 5<sup>th</sup> structure using combinational optimal controller in master and slave subsystems

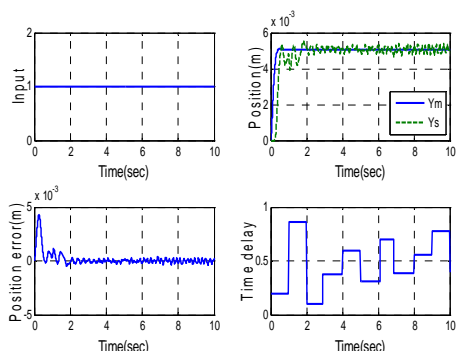


Fig. 9: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the 6<sup>th</sup> structure using optimal controller and wave variable transformation

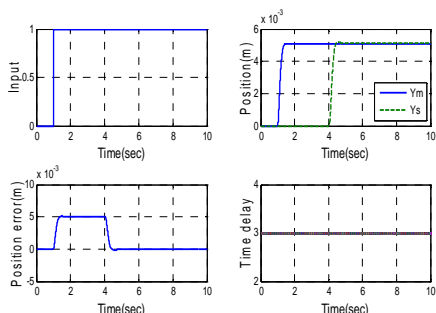


Fig. 10: simulation of master-slave behaviors (a: input; b: position; c: position error; d: time delay) for the 7<sup>th</sup> structure using optimal controller and wave variable transformation

## V. CONCLUSION

In this paper, we have proposed the design of a bilateral controller for teleoperation systems with variable time delay. These controllers ensure the passivity, transparency and high efficiency of the system and take into account the communication delays and the interaction between the slave and the environment. In these structures, the optimal controller receives delayed position signals of master and slave and compares them to the reference signals and applies control signals to the master and slave in a way that the tracking error between them is minimized and they can track each other in the least possible time. For stabilizing the system for variable time delay, wave variable method has been used. The simulation results have shown the superiority of the augmented technique to the commonly used wave-variable method. Simulation results have shown excellent performance in motion tracking and good response in force tracking. The model uses a state space representation of the teleoperation systems considering all the possible interaction that can appear in the operator-master-slave-environment set. Comparative experiments demonstrate the validity of the proposed controls method and their excellent performance of motion/force tracking.

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